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Interaction of solitons with a strong inhomogeneity in a nonlinear optical fiber

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We consider the interaction of fundamental and higher-order solitons with a strong localized inhomogeneity of the dispersion or nonlinear coefficient in a nonlinear optical fiber. The inhomogeneities are modeled by δ functions in the corresponding nonlinear Schrödinger equations, but they are *not* treated as small perturbations. Actually, they could also represent long pieces of, respectively, (i) linear or (ii) zero-dispersion fiber inserted into the bulk fiber, which is of wider interest for applications than the inhomogeneities proper. We obtain analytically a pulse transformed by the inhomogeneity and then calculate its soliton content by means of numerical solution of the corresponding Zakharov-Shabat equations. It is found that an inhomogeneity in the nonlinearity can split the incoming fundamental soliton (1-soliton) into a symmetric pair of separating small-amplitude solitons, although the larger part of the initial energy will be lost in this case in the generation of dispersive waves (continuous radiation). In contrast to this, an inhomogeneity in the dispersion only attenuates the output soliton but never splits it. We obtain a more interesting result upon considering the interaction of 2-solitons with the nonlinearity inhomogeneity: a moderately strong inhomogeneity can split the 2-soliton into a symmetric pair of separating fundamental solitons, generating only a fairly small amount of dispersive waves. (Here, an n soliton is a solitary pulse with an amplitude n times that of a soliton but with the same width.) This result may be important for applications, as it is well known that the soliton lasers generate pulses in the form of 2-solitons. We thus predict that an inserted piece of the zero-dispersion fiber can effectively transform 2-solitons into equal and separating fundamental solitons of good quality. For comparison, we also consider the same problem for the 1.5- and 2.5-solitons; in the latter case, a third soliton may be generated. The results obtained can be applied as well to nonlinear waveguides of other physical origin, e.g., guides for internal waves in the ocean.

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I. INTRODUCTION

A recent development in technology allows one to fabricate silica fibers with a well-controlled variable cross section [1]. These fibers can be used as nonlinear lightguides with a variable dispersion coefficient, which is why they were given the name "dispersion-decreasing fibers" (DDF's). As early as 1987, it had been proposed to use DDF's for the pulse compression [2]. Experiments with ultrashort (subpicosecond) optical pulses launched into the DDF [3] have demonstrated that they indeed provide an effective compression of the pulses, practically without disturbing their solitonic shape.

However, DDF's can be used for more sophisticated applications than simple pulse compression, e.g., for the improved soliton reshaping in a very long lossy optical cable [4] and for the high-repetition-rate generation of an array of solitons from a continuous wave [5]. Another interesting application is the propagation of an ultrashort soliton in a fiber with a periodically modulated dispersion coefficient. This problem was considered semianalytically in Ref. [6] by means of the variational approximation, which was first developed for the model of the homogeneous (constant-dispersion) fiber [7] and later extended to the DDF in Ref. [8]. There it was predicted that, for a given initial energy of the soliton, there would exist a critical value of the modulation amplitude beyond which the soliton would be suddenly destroyed after it had propagated for a certain period. Very recently, direct systematic simulations of this model have corroborated this prediction qualitatively and, in a part, even quantitatively [9] (see also Ref. [10]). However, another fundamental fact revealed by these simulations is that the above-mentioned destruction mode does not actually destroy the soliton. Rather the soliton splits into two secondary solitons, accompanied by a burst of dispersive waves (continuous radiation). (This particular decay mode could not have been modeled with the simple ansatz employed in Ref.

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[6]. The only destructive mode allowed by that ansatz was, as a matter of fact, a sudden decay of the soliton into radiation.)

The above works were concerned with the physics of solitons when very gradual changes occurred. Here we shall look at the extreme opposite case when such changes are very sharp. Perhaps surprisingly, we find some similarities (as well as differences), indicating perhaps some universal behavior of solitonlike systems. Also, the physics of these processes is just as interesting as in the gradual case. Here we will consider the opposite limit: the interaction of a soliton with a strongly localized inhomogeneity. First, we consider an inhomogeneity in the fiber's nonlinear (Kerr) coefficient and then an inhomogeneity of its dispersion coefficient. In each case, we can allow the inhomogeneity to be strong such that one would not expect perturbative expansions in the strength of the inhomogeneity to be valid. These problems are of interest in themselves since they represent a fundamental type of dynamical behavior in real nonlinear inhomogeneous optical fibers and, more generally, in real inhomogeneous nonlinear waveguides of an arbitrary physical nature (one example of which is the natural waveguides for internal gravity waves in the ocean [11]). Naturally, it will also be feasible to observe these interactions experimentally. Indeed, a localized inhomogeneity of the dispersion in the optical fiber can easily be created by means of the above-mentioned technique (see Ref. [12], in which it was demonstrated that, in real tapered silica fibers, a dependence of the effective dispersion coefficient upon the fiber's diameter can sometimes be extremely sharp; thus one then has a strongly localized inhomogeneity). Similarly, an inhomogeneity of the Kerr coefficient can also be induced by tapering the fiber [12]. Alternatively, an inhomogeneity in the Kerr coefficient can be fabricated by simply doping a segment of the fiber with a resonant dopant, tuned to the carrier wave length of the soliton (see, e.g., Ref. [13]).

The problem considered in this work has actually a more general purport, as it may be applied to inhomogeneous nonlinear wave guides of an arbitrary physical nature. A well-known example different from optical fibers is furnished by natural guides for internal waves in the ocean [11]. These waveguides are quite inhomogeneous and the guided internal waves are frequently nonlinear [11].

Returning to the optical-fiber model, the same perturbation has an alternative physical interpretation, which is actually essentially more important than the simple local inhomogeneity. This alternative interpretation is simply that perturbations of the nonlinear and dispersion coefficients proportional to the δ function are equivalent to arbitrarily long pieces of the, respectively, (i) zero-dispersion (purely nonlinear) and (ii) purely linear fibers inserted into the standard bulk fiber. If the inserted piece is long enough, the equivalent δ-like perturbation will be strong, but it will keep the same functional form. As will be shown in this work, the former case, i.e., an inserted segment of the dispersion-shifted fiber with a nearly zero value of the effective dispersion coefficient, may have a real application in photonics: it can readily transform a 2-soliton into a *symmetric* pair of separating fundamental solitons (1-solitons) with a very low level of radiative losses. This application is important because, as is well known (see [14] and references therein), a soliton laser, which is a standard soliton source, usually produces 2-soliton (an n soliton is a solitary pulse with an amplitude n times that of a soliton, but with the same width as the soliton), while one needs fundamental solitons for the use in the soliton-based communication lines and logic chips. Thus one can employ the above technique to split each 2-soliton into a pair of *equal* fundamental solitons. Besides that, it is relevant to mention that the study of soliton dynamics in nonlinear fibers operating near the zero-dispersion point is an interesting subject in itself [14].

The remainder of the paper is organized as follows. In Sec. II we start with the analysis for the case when the coefficient of nonlinearity has a local inhomogeneity approximated by the δ function, with the incoming pulse being a fundamental soliton. Outside of the inhomogeneity region (which is formally a single point), the system is governed by the nonlinear Schrödinger (NLS) equation. Using this approximation, we then find the exact solution of the evolution equation in the infinitesimal vicinity of the inhomogeneity. Thus, given an input pulse, we can exactly determine the output pulse. When the output pulse reenters the bulk fiber, it will not be a pure soliton. So we apply the direct scattering transform to the output pulse in order to determine its soliton and radiation content. We find that, when the input is a fundamental soliton, the amplitude of the output soliton rapidly decreases with increasing inhomogeneity strength and, at a critical point, it *bifurcates.* Beyond this critical point, a pair of symmetric separating solitons appears. With increasing strength of the inhomogeneity, their amplitude initially increases slightly, reaching a rather small maximum value, but then quickly decays with further increase of the strength.

In Sec. III we consider the same problem for a localized inhomogeneity in the dispersion. In this case, a single output soliton is always produced. Its amplitude gradually decays as the strength of the inhomogeneity is increased. In both cases, the eventual result is independent of the sign of the inhomogeneity. (The reason for this is that a δ-function-like inhomogeneity always dominates all other coefficients.)

In Sec. IV we return to the case where there is an inhomogeneity in the nonlinearity, but this time we consider more general input pulses than a single fundamental soliton. As was mentioned above, a particularly interesting problem is when the input pulse is a 2-soliton pulse. When such a pulse passes through the inhomogeneity, the interaction with the inhomogeneity changes the eigenvalues of the two constituent solitons, causing the imaginary parts of the eigenvalues to move closer together. There is a critical value of the nonlinear inhomogeneity parameter (this value is not very large, being close to 0.5), at which a *bifurcation* takes place. When this occurs, the imaginary parts of the two eigenvalues become equal (and remain equal beyond the bifurcation point) and simultaneously the real parts of the eigenvalues become equal in magnitude but opposite in sign. Thus the former 2-soliton pulse is split into two separating fundamental solitons. Now there is a crucial difference between this bifurcation and that for an input consisting of only a single fundamental soliton. This process for a single fundamental soliton is accompanied by strong generation of dispersive waves. However, for the 2-soliton pulse, at the bifurcation point, the energy lost in the generation of radiation is not more than 0.5%. This can have some very important applications in photonics. Even at a value of the inhomogeneity parameter that is twice as large as that at the bifurcation point, the share of the radiative losses still stays below 10%. We then extend the analysis to consider 1.5- and 2.5-solitons. For the case of the 1.5-soliton, the results prove to be quite similar to those obtained for the 2-soliton, more so than that for the fundamental soliton. On the other hand, for the 2.5-soliton, we find that not only do we have a similar bifurcation, but also a new feature appears: namely, the generation of a third soliton of a very small amplitude. Based on these results, we can conclude that the splitting of a 2-soliton state into two separating soltions should be quite stable against variations of the input.

II. INTERACTION OF THE FUNDAMENTAL SOLITON WITH AN INHOMOGENEITY IN THE NONLINEARITY

We will start the analysis from the case when the nonlinear (Kerr) coefficient of the fiber has a strong localized inhomogeneity. The simplest model of this system is the NLS equation with a perturbation term proportional to the δ function:

$$iu_z + u_{\tau\tau} + 2|u|^2 u = -\epsilon |u|^2 u\, \delta(z)\,. \tag{1}$$

Here, as usual, z and τ stand for, respectively, the propagation distance and the retarded time, $u(z,\tau)$ is the complex envelope of the propagating waves, and proper nondimensionalization of all the coefficients in the NLS equations is implied.

If the perturbation on the right-hand side of Eq. (1) is interpreted literally as a local inhomogeneity, using the δ function implies that an actual length of the segment with the altered value of the nonlinear coefficient is much smaller than the dispersion length of the soliton (which is also called the soliton period). In nonlinear silica fibers, for instance, typical subpicosecond solitons, which should be appropriate candidates for the experimental realization of this problem, can have soliton periods on the order of hundreds of meters. Thus a segment of the fiber no longer than several dozen meters, doped (or changed otherwise) so as to have a different Kerr coefficient, can be described by Eq. (1) with the δ function perturbation.

On the other hand, the same perturbation on the right-hand side of Eq. (1) effectively describes an inserted segment of a dispersion-shifted fiber, provided its effective dispersion coefficient is negligibly small and the higher-order dispersion [14] can be neglected also. Let the length of the inserted segment be L and its Kerr coefficient be K. Then it is straightforward to see that insertion of such a segment is exactly equivalent to adding the perturbation term on the right-hand side of Eq. (1) with

$$\epsilon \equiv KL\,. \tag{1$'$}$$

Proceeding, it is necessary to stress that we do not assume the perturbation parameter ϵ to be small and, accordingly, the perturbation is not assumed to be small or weak in any sense. We will find, both in this section and later in Sec. IV, that we can obtain results valid for arbitrary (nonsmall) values of ϵ. To fully justify using the model with the δ function, one should, strictly speaking, verify that the results obtained for the δ function are equivalent to a limit of a narrow but finite-length perturbation when the length tends to zero. This can be done easily.

In this section, the input pulse will be taken to be a fundamental soliton

$$u_-(\tau) = 2\eta\, \mathrm{sech}(2\eta\tau)\,, \tag{2}$$

where η is the amplitude of the soliton (in the fiber optics, a more important physical characteristic is the soliton's peak power $4\eta^2$) and the subscript "$-$" implies that the soliton is taken at $z = 0^-$, i.e., just before interacting with the δ-like inhomogeneity. The soliton at $z = 0^-$ can be given an additional parameter, viz., a central frequency ω, so that the wave form (2) is replaced by

$$u_-(\tau) = 2\eta\, \mathrm{sech}(2\eta\tau)\, e^{-i\omega\tau}\,. \tag{2$'$}$$

However, because the perturbed NLS equation (1) remains Galilean invariant, we may transform to the Galilean frame where ω is zero, perform the analysis, and then transform back. The net result is that we may ignore the parameter ω.

Straightforward integration of Eq. (1) over an infinitesimal vicinity of the point $z = 0$ (obviously, only the first term on the left-hand side and the perturbation term on the right-hand side remain important in this vicinity) yields the following for the output wave form at $z = 0^+$:

$$u_+(\tau) = 2\eta\, \mathrm{sech}(2\eta\tau) \exp\left[-4i\epsilon\eta^2 \mathrm{sech}^2(2\eta\tau)\right]. \tag{3}$$

The output wave form (3) is a combination of soliton(s) and continuous radiation. The latter would separate away from the soliton and disperse away as the output pulse reenters and propagates down the bulk fiber. Thus the most physically interesting issue is to find soliton content of the output. A straightforward way to solve this problem is to apply the inverse scattering transform (IST) [15] to the output pulse. As is well known, the IST for the NLS equation is based on the auxiliary linear Zakharov-Shabat (ZS) equations for a two-component Jost function $\left(\psi^{(1)}, \psi^{(2)}\right)$:

$$\psi_\tau^{(1)} + i\lambda\psi^{(1)} - u\psi^{(2)} = 0\,, \tag{4a}$$

$$\psi_\tau^{(2)} - i\lambda\psi^{(2)} + u^*\psi^{(1)} = 0\,, \tag{4b}$$

where λ is the spectral parameter and the asterisk stands for the complex conjugation.

Inserting a given wave form [e.g., that given by Eq. (3)] into Eqs. (4), one would search for discrete eigenvalues of λ lying in the upper half of the complex plane. Each eigenvalue corresponds to a soliton that will asymptotically (as $z \to \infty$) separate out from the given initial wave form. The imaginary and real parts of the eigenvalue are proportional, respectively, to the soliton's amplitude and central frequency.

For the output wave form (3), the ZS equations cannot be solved analytically. However, a numerical solution is possible in this case. Implementing the numerical solution, we will concentrate on finding the complex eigenvalues of λ. The imaginary and real parts of the eigenvalue(s) will be functions of two parameters, viz., the input soliton's amplitude η and the inhomogeneity strength ϵ. However, using the scale invariance of Eq. (1), one may set $\eta \equiv 1$, which we will do. Thus we are left with only the single nontrivial parameter, ϵ.

In Fig. 1 we display the dependence of the amplitude and central frequency of the soliton(s), found in the output wave form by means of the numerical solution of the ZS equations (4) vs the perturbation strength ϵ. In the region $0 < \epsilon < 0.6$, we find exactly one soliton with a zero central frequency (obviously, if the soliton is single, its central frequency must be zero due to the symmetry of the problem). The soliton's amplitude monotonically decreases with increase of ϵ, vanishing at $\epsilon \approx 0.6$. With the decrease of the soliton's amplitude, the remaining part of the initial energy appears as continuous radiation generated in the output wave field.

At $\epsilon \approx 0.6$, a *bifurcation* takes place: at the same value of ϵ (close to 0.6) at which the amplitude of the soliton vanishes, a pair of new symmetric solitons appears with an infinitesimal amplitude. Beyond the bifurcation point, these solitons each have a nonzero central frequency (due to the symmetry, the amplitudes of the two solitons are exactly equal, while their central frequencies are exactly opposite). With a further increase of ϵ, the central frequencies monotonically increase, while the amplitudes attain a rather small maximum value (less than 10% of that of the input soliton) at $\epsilon \approx 0.8$ and then decrease again, vanishing for all practical purposes beyond $\epsilon > 1.0$.

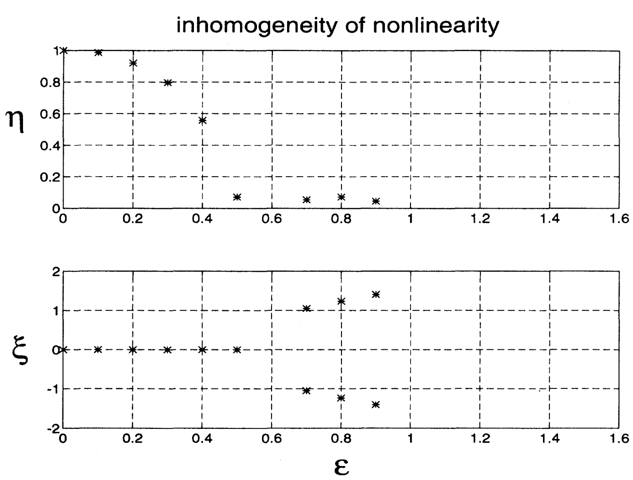

FIG. 1. Characteristics of the output solitons produced from the input fundamental soliton by the nonlinear perturbation vs the perturbation strength ϵ: the amplitude of the output soliton(s) (the upper portion of the plot) and their central frequencies (the lower portion). The central frequency ω defined as per Eq. (2) is twice the quantity displayed in the lower portion of Fig. 1.

This bifurcation is different from the one analyzed recently in the context of another problem, viz., the splitting of a pulse governed by the unperturbed NLS equation into two solitons under the action of a chirp, i.e., a phase with a variable frequency [16]. In that case, it was shown that the amplitude of the soliton produced by an initial box-shaped pulse decreased with increase of the chirp, but, at a certain point, a new soliton was produced. The new soliton appeared with a zero amplitude. With further growth of the chirp, its amplitude increased while that of the original soliton kept decreasing until the two amplitudes merged at the bifurcation point. Beyond this point, the amplitudes remained equal, but the solitons had opposite central frequencies.

Another feature of this problem is also noteworthy. Generally speaking, opposite signs of ϵ in Eq. (1) are not equivalent. However, the results obtained in this section do not depend upon the sign of ϵ. Indeed, as it immediately follows from Eq. (3), the change of the sign amounts to complex conjugation of $u_+(\tau)$. Next, it follows from the ZS equations (4) that, in turn, the complex conjugation of u leads to the following change of the eigenvalues: $\lambda(u^*) = -\lambda^*(u)$. So we conclude that changing the sign of ϵ does not affect the amplitude(s) of the resultant soliton(s), which are proportional to the imaginary part of the eigenvalue, but reverses the sign of the central frequency, which is proportional to the real part. However, the latter change is immaterial since we always obtain either a single soliton with a zero central frequency or two symmetric solitons with equal but opposite central frequencies.

The results presented in this section are universal in the sense that they completely describe all possible interactions of the fundamental soliton with the δ-function-like inhomogeneity of the Kerr coefficient. In Sec. IV, it will be demonstrated that the transformation of a higher-order soliton under the action of the the same perturbation leads to altogether different results.

III. INHOMOGENEITY IN THE DISPERSION

The simplest model taking into account a localized inhomogeneity in the dispersion coefficient has the form [cf. Eq. (1)]

$$iu_z + u_{\tau\tau} + 2|u|^2 u = -\epsilon u_{\tau\tau}\delta(z)\,, \qquad (5)$$

where again, the perturbation coefficient ϵ is actually a product of the change of the local dispersion coefficient and the length of the segment with the altered dispersion coefficient. An alternative realization of this perturbation is a segment of a purely linear fiber of length L, with a dispersion coefficient D, spliced into the bulk fiber. It is easy to see that this is equivalent to the term on the right-hand side of Eq. (5) with $\epsilon \equiv DL$; cf. Eq. (1′). The results we obtain below are less interesting than the above: we will see simply a gradual degradation of the soliton with increase of ϵ and no bifurcation will take place. Like in the preceding section, we do not assume the parameter ϵ to be small and it can be easily demonstrated that the results to be obtained below for an arbitrary value of ϵ are equivalent to the limit that can be obtained from a regularized perturbation expansion with a finite support when the length of the support tends to zero.

The input pulse is again taken to be a fundamental soliton (2). If it is taken in the more general form (2′), the transformation

$$u(z,\tau) \equiv \tilde{u}\,(z,\tilde{\tau}) \exp\left[-i\omega\tau - i\omega^2 z + i\epsilon\omega^2\theta(z)\right], \quad (6)$$

$$\tilde{\tau} \equiv \tau + 2\omega z - 2\epsilon\omega\theta(z)\,, \quad (7)$$

where $\theta(z)$ is the step function, brings the input soliton back into the form (2); according to Eqs. (6) and (7), a nonzero central frequency of the input soliton gives rise to shifts in the phase and position of the output pulse, but, obviously, it does not alter its soliton content.

To find the output pulse in the present case, Eq. (5) should be solved in an infinitesimal vicinity of the point $z = 0$ by means of the Fourier transformation. Before doing this, we can again set $\eta \equiv 1$ for the input pulse, using the scale invariance of Eq. (5). Finally, the output pulse can be obtained in the integral form produced by the inverse Fourier transformation:

$$u_+(\tau) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\tau' e^{i\omega(\tau-\tau')+i\epsilon\omega^2}\,\mathrm{sech}(2\tau')\,, \quad (8)$$

which in general is complex. The shape of the real part of the output pulse (8), obtained by the numerical computation of the integral (8), is displayed in Fig. 2 for $\epsilon = 1$.

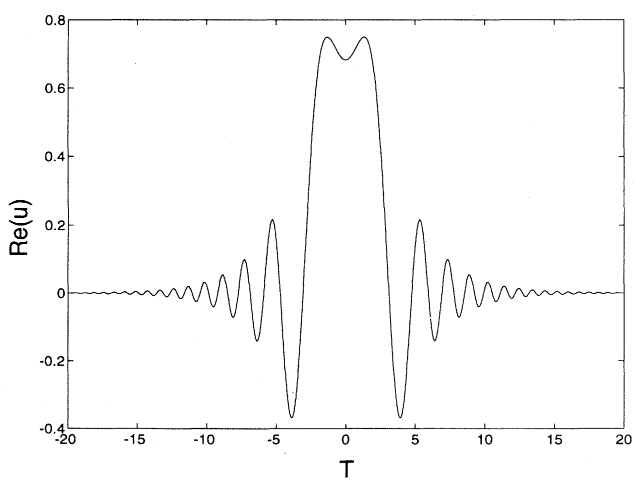

FIG. 2. Shape of the output pulse (8) produced from the input fundamental soliton by the dispersion inhomogeneity at $\epsilon = 1$.

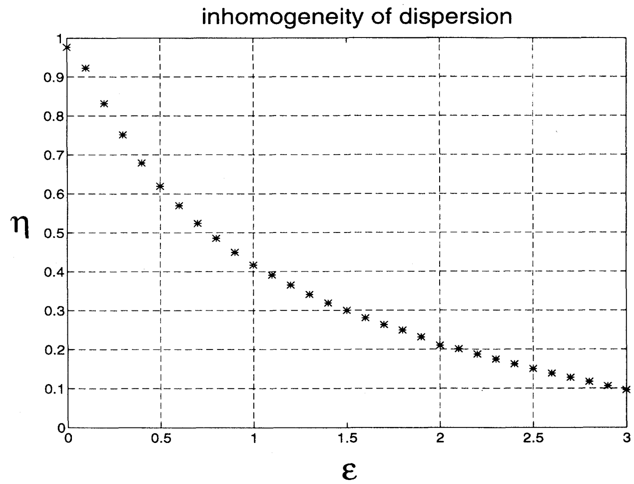

FIG. 3. Amplitude of the output soliton produced from the input fundamental soliton by the dispersion inhomogeneity vs the perturbation strength ϵ.

Solving the ZS equations (4) with the potential $u_+(\tau)$ defined by Eq. (8), we have that, in contrast to the case considered in Sec. II, the output pulse always contains only a single soliton plus dispersive waves, with the soliton's central frequency equal to zero. The dependence of the amplitude of this soliton upon the inhomogeneity parameter ϵ is displayed in Fig. 3. The amplitude slowly and smoothly goes to zero as ϵ increases.

IV. TRANSFORMATION OF A HIGHER-ORDER SOLITON BY AN INHOMOGENEITY IN THE NONLINEARITY

In this section, we will return to the model (1), assuming that the inhomogeneity is produced by an inserted segment of a zero-dispersion nonlinear fiber according to Eq. (1′). However, unlike what was done above in Sec. II, here we will consider input pulses different from the fundamental soliton (2):

$$u_-(\tau) = 2N\,\mathrm{sech}(2\tau)\,, \quad (9)$$

where we have set, as above, $\eta \equiv 1$ [cf. Eq. (2)] and the number N is usually called the order of the higher-order soliton [14]. As mentioned in the Introduction, the case $N = 2$, i.e., the so-called 2-soliton state, is of special interest for applications because the soliton lasers, which are standard sources of high-quality solitons, usually generate pulses exactly in the form of the 2-solitons [14].

The subsequent procedure is quite similar to that employed in Sec. II: one can first easily solve Eq. (1) in the infinitesimal vicinity of the point $z = 0$ [see Eq. (3)] and then the soliton content of the resultant output pulse can be found numerically by means of the ZS equations (4). The results are displayed in Fig. 4.

For the N-soliton (9), the optical energy (number of photons)

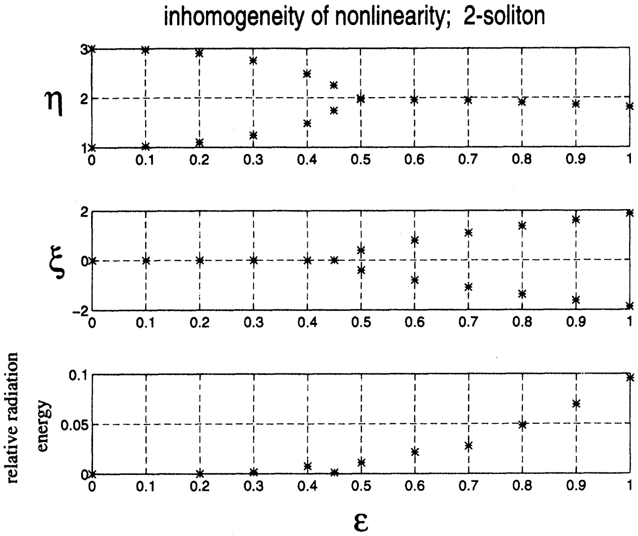

FIG. 4. Characteristics of the output solitons produced from the input 2-soliton by the nonlinear perturbation. The upper and central portions have the same meaning as in Fig. 1; the lower portion displays the share of the radiation energy vs the perturbation strength ϵ.

$$E \equiv \int_{-\infty}^{+\infty} |u(\tau)|^2 d\tau \tag{10}$$

takes the value $4N^2$. In particular, for the 2-soliton the energy is 16. On the other hand, each discrete eigenvalue with the imaginary part η corresponds to a fundamental soliton carrying the energy 4η. Looking at the upper portion of Fig. 4, one notices that, at $\epsilon = 0$, the two values of η are 3 and 1, so that their net energy is indeed equal 16.

With an increase of ϵ, the two eigenvalues get closer, remaining purely imaginary. This is accompanied by the generation of some continuous radiation. The share S of the total energy lost to continuous radiation can be calculated, according to the above formulas, as $S = 1 - (\eta_1 + \eta_2)/N^2$, where η_1 and η_2 are the amplitudes of the output fundamental solitons. The lower portion of Fig. 4 displays the share S vs the perturbation parameter ϵ. Comparing this plot to Fig. 1, one immediately notices a crucial difference between the present bifurcation and that considered in Sec. II. In Sec. II, the radiative losses dominated; now they are fairly small.

The bifurcation takes place at the critical value

$$\epsilon_{\text{cr}} \approx 0.47. \tag{11}$$

Beyond the bifurcation point, the amplitudes of the two solitons remain strictly equal, while they acquire equal but opposite central frequencies, which, as well as in the case of the bifurcation corresponding to Fig. 1, implies that the two solitons are separating at a certain velocity. Thus the present bifurcation transforms the input 2-soliton into a pair of well-separated fundamental solitons. As mentioned in the Introduction, this process is of definite interest for applications, as it allows one to transform a higher-order soliton (say a 2-soliton produced by the soliton laser) into a pair of fundamental solitons. This transformation may be accomplished by a very simple means, viz., a piece of a zero-dispersion fiber spliced into the bulk fiber. The necessary length of the inserted piece can be readily found from Fig. 4 and Eq. (11), using the relation (1′). A very important asset of this transformation is the fact that, according to the data displayed in the lower portion of Fig. 4, the radiative losses may be kept to a very low level. In particular, exactly at the bifurcation point, the share of the radiation energy is not more than 0.5%. Even at the value of ϵ, which is twice the critical value (11), the radiation absorbs less than 10% of the total energy. Thus this is a very efficient means of generating two solitons of *equal* amplitude from a single 2-soliton pulse.

We have also considered the same problem for some noninteger values (1.5 and 2.5) of the order N of the input soliton. These provide information on the stability of the above results for the 2-soliton case. In Fig. 5 we display the results when the input is a 1.5-soliton. The unperturbed pulse corresponding to $N = 1.5$ contains, as follows from the well-known results of Satsuma and Yajima [17], exactly one eigenvalue with $\eta = 2$, which agrees with what is shown in Fig. 5 at $\epsilon = 0$. The main features of variation in the output with increase of ϵ are clearly seen in Fig. 5. It is noteworthy that this case, being intermediate between $N = 1$ (Fig. 1) and $N = 2$ (Fig. 4), seems essentially closer to the latter case. In particular, the bifurcation takes place at ϵ quite close to the critical value (11). It is also interesting to

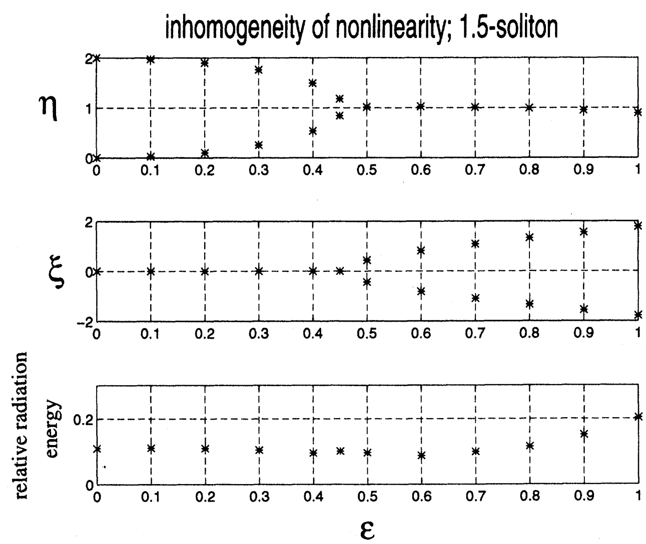

FIG. 5. Characteristics of the output solitons produced from the input 1.5-soliton by the nonlinear perturbation. The upper, central, and lower portions have the same meaning as in Fig. 4.

compare the radiative losses in the case $N = 1.5$ with those corresponding to $N = 1$ and $N = 2$. As seen from the lower portion of Fig. 5, the share of the radiation energy is close to 10% at the bifurcation point, while it is much larger at $N = 1$ (Fig. 1) and much smaller at $N = 2$ (Fig. 4).

Figure 6 displays the results for the 2.5-soliton. Notice, first of all, that, according to the Satsuma-Yajima formulas [17], the unperturbed 2.5-soliton contains exactly two eigenvalues with $\eta_1 = 4$ and $\eta_2 = 2$, which agrees with Fig. 6 at $\epsilon = 0$. (Actually, since 2.5 is a critical value, a third soliton must appear just above 2.5. Thus, at 2.5, strictly speaking, we also have a third soliton, but with a zero amplitude.) At $\epsilon > 0$, these two eigenvalues approach each other, while a third small-amplitude soliton now appears in the output (depicted by the dashed line in Fig. 6). An interesting feature of these results for $N = 2.5$ are the unresolved oscillations in the radiation energy share vs the perturbation parameter ϵ [see Fig. 6(c)].

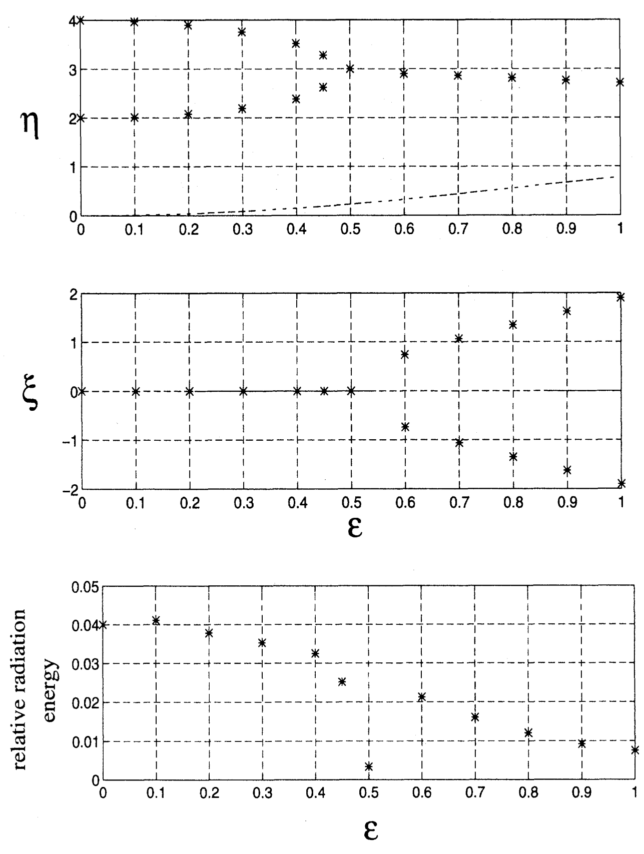

FIG. 6. Characteristics of the output solitons produced from the input 2.5-soliton by the nonlinear perturbation: the amplitudes and central frequencies of the output solitons (shown, respectively, in the upper and middle portions of the plot) vs the perturbation parameter ϵ; the share of the radiation energy (the middle lower portion) vs ϵ. In the plots, the dashed curve shows the characteristics of the third output soliton.

To conclude this section, note that the 2-soliton pulse, being an unstable solution to the unperturbed NLS equation, can be easily split by almost any perturbation into two fundamental solitons, with final amplitudes close to 3 and 1, which correspond to the two eigenvalues in the unperturbed 2-soliton pulse. An example of this is found in recent work [18] where this type of a splitting was simulated within the framework of the NLS equation by using a periodically modulated dispersion coefficient (cf. the work [6]). The principal difference between that and the splitting mechanism considered here is that our mechanism performs a "deep processing" of the input 2-soliton pulse, splitting it into two symmetric fundamental solitons with *equal* amplitudes.

V. CONCLUSION

In this work, we have considered the transformation of fundamental and higher-order solitons in a nonlinear optical fiber under the action of two fundamental types of local inhomogeneities, viz., inhomogeneities in the nonlinearity and in the dispersion. In both cases, the inhomogeneities were treated by using the assumption that the length of the inhomogeneous section was much smaller than the soliton's dispersion length, so that the inhomogeneity could be approximated by a δ function, but at the same time, we did not assume the inhomogeneity to be a small perturbation. We have also considered an alternative (and more important physically) interpretation of the same perturbations, in the form of arbitrarily long pieces of (i) zero-dispersion or (ii) purely linear fiber spliced into the bulk fiber. The adopted model was based on the NLS equation with an additional term representing the local inhomogeneity. The solution of the problem was obtained by the exact integration of the evolution equation across the inhomogeneity and then application of the IST to the resultant output pulse in order to find its soliton content. Using the scaling invariance of the models, in each case we were able to bring them into a single-parameter form. We found that the inhomogeneity of the nonlinear (Kerr) coefficient can cause the input fundamental soliton to split into two small-amplitude secondary solitons; however, most of the initial energy would be expended on the generation of radiation. In contrast to this, the dispersion inhomogeneity never splits the incoming fundamental soliton, but merely induces a partial decay into continuous radiation. In both cases, although the full equations are not invariant with respect to change of sign in front of the inhomogeneous term, the obtained results do not depend upon this sign.

Consideration of 2-soliton input pulses with a nonlinear inhomogeneity yields altogether different results. It was demonstrated that, provided the perturbation strength ϵ exceeded a critical value that is not really large, this perturbation effectively transforms the 2-soliton into a pair of fundamental solitons with equal amplitudes and with the radiative losses being very small. Approximately the same result can be obtained with 1.5-

and 2.5-soliton pulses. In particular, the 2.5-soliton input pulse contained an additional (third) small-amplitude soliton. Thus one would want to keep $N < 2.5$. This result is of definite interest for applications since it opens an effective way to transform N-soliton pulses (N close to 2) into two fundamental standard solitons.

One could also consider a more general case, wherein one would combine the local inhomogeneities of both the nonlinearity and dispersion (note that tapering a real optical fiber is apt to produce such a combined inhomogeneity [12]). However, now the treatment of this problem is no longer as simple as has been done here. The above two cases reduce to linear equations. The general combined case is a fully nonlinear case. One can easily see that by the following argument. If we interpret the perturbation as an inserted segment of fiber with arbitrary parameters, it now has both dispersion and nonlinear coefficients. Thus, in this small segment, a pulse would propagate according to a NLS equation, however, with *different* coefficients. Thus a soliton in the bulk fiber will not be a soliton in the inserted segment. In particular, any pulse put into the segment will have, in terms of the IST, different scattering data in the segment than in the pulse. Thus, in the segment, a different propagation will occur. However, in the segment, the pulse will propagate according to a NLS equation, although with different coefficients. Thus one must use the full IST for this new NLS in order to determine the propagation inside the segment. Then, at the end of the segment, one must take the output pulse and decompose it according to the IST for the NLS equation in the bulk fiber, in order to determine the soliton content that will exist upon reentering the bulk fiber. Clearly, these more general results will now be a function of three parameters (the dispersion and nonlinearity of the segment, as well as the segment length). This will be much more complex than the two limits considered here (and is also clearly beyond the scope of this work). Of course, in this case the possible effects could be more complex. However, these effects would still have to approach the results obtained here in the appropriate limits.

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